Conditional Value-at-Risk Model for Optimizing Crop Insurance Under Climate Variability

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Abstract

This paper studies the application of the Conditional Value-at-Risk (CVaR) model to the crop insurance industry under climate variability. We designed the model to help farmers compare various crop insurance products in order to reduce climate and price risks. The objective is to minimize farmers' return losses while using CVaR to control the risk aversion level. We illustrate the application of the model by studying a farm with two crops – cotton and peanut – in Jackson County, FL. Crop insurance contracts with the greatest expected return were: for peanut, 75% actual production history (APH) during El Niño and Neutral years, and 65% APH during La Niña years; and for cotton, 75% APH in all El Niño Southern Oscillation phases. Risk-averse farmers might also select 75% APH for peanut during La Niña years.

Introduction

Farmers face climate and market risks that are beyond their control. There are numerous crop insurance products farmers could purchase to reduce these risks. Consequently it is meaningful to optimize the farmers' crop insurance selection.

Crop production depends heavily climate conditions, which in turn are affected by phase of the El Niño Southern Oscillation (ENSO) phenomenon. In the Southeast USA, ENSO effects on climate are well documented (Ropelewski and Halpert, 1986; Rogers, 1988; Sittel, 1994; Green et al., 1997). El Niño effects in the southeastern USA are strongest during winter and spring, bringing more rainfall and cooler temperatures. La Niña brings warmer and drier winters. Recent advances in climate forecasting and the consequent ability to predict climate fluctuations provide opportunities to improve the management of climate-associated risks in agriculture (Hansen et al., 1998). Use of ENSO-based climate forecasts has been shown to help reduce risks faced by agricultural enterprises (Hansen, 2002; Jones et al., 2000). Fraisse et al. (2005) demonstrated the ability to use ENSO-based climate forecast combined with crop growth models to improve crop insurance strategy.

Crop insurance is a major component of risk management that farmers could use together with climate information to optimize their risk-return characteristics (Changnon et al., 1999). Few studies have explored the interactions between common crop insurance contracts and the farm value of ENSO-based forecasts (Cabrera et al., 2005a; Mjelde and Hill, 1999; Mjelde et al., 1996). Cabrera et al. (2005b) presented a systematic study to strategize the selection of crop insurance products in a whole farm portfolio under climate variability. They analyzed risk associated with each ENSO phase, based on long series of synthetic crop yields and independent synthetic commodity prices. Their objective was to maximize the farmer's expected utility function for different risk aversion levels. Cabrera et al. (2005b) modeled and identified optimal planting dates and crop insurance products. Utility function models, however, are hard to implement and calibrate because of their conceptual complexity.

A Conditional Value-at-Risk (CVaR) measure has some attractive properties over the utility function. First, CVaR specifies risk preference in simple monetary terms with a confidence level. Thus farmers can easily select their level of personal risk. For example, the statement "90% CVaR must be less than $100" means the average of the worst 10% outcomes must be less than $100. Second, CVaR is a statistical characteristic depending upon the distribution of outcomes, so it can model risk aversion levels without the utility function. Third, CVaR is very similar to Value-at-Risk (VaR), percentile of loss distribution, which is a standard measure used in various engineering applications. Fourth, CVaR is a *coherent measure of risk,* as defined by Artzner et al. (1999), with axiomatic-mathematical properties desirable for a perfect risk measure. Fifth, Rockafellar and Uryasev (2000) showed that CVaR of a discrete random variable is a convex piece-wise linear function that can be optimized with linear programming. Sixth, CVaR is more conservative than VaR. By definition $\mathrm{CVaR} \geq \mathrm{VaR}$ because CVaR emphasizes outcomes in the tail of a distribution that are beyond VaR. Because of these advantages, CVaR is an exceptional risk measure and it is gaining popularity in various applications, especially in finance.

The main goal of this study is to design a model to help farmers evaluate alternative crop insurance products in order to reduce climate and price risks according to realistic risk aversion levels included in the CVaR function. In addition to the optimal crop insurance selection, the model would help farmers to allocate land to different planting dates for the included crops. The model is applied to a cotton-peanut farm in Jackson Co., FL.

Model

The major difference between this study and that of Cabrera et al. (2005b) is that we use the Conditional Value-at-Risk (CVaR) measure (Rockafellar and Uryasev, 2000, 2002), instead of the utility function to model farmer risk preferences. In addition, this study generates commodity prices based on historical ENSO records with a Monte Carlo simulation.

Analyses were performed for every ENSO phase separately with the objective to minimize loss return under a given CVaR constraint. The number of scenarios equals the number of possible yields and market prices using historical data. Decision variables include the land area allocated by planting date and the crop insurance products selected.

2.1 Notations

A farm grows crops $k = 1, 2, \ldots, K$ on areas q_k, $k = 1, 2, \ldots, K$ allocated for each crop.

Planting dates for crop k are indexed as d_k and scenarios are indexed as $s = 1, 2, \ldots N$, are historical records for each ENSO phase. Crop insurance contracts are indexed by $i = 1, 2, \ldots, I$. Parameters used for each outcome are listed in Table 1.

The decision variables are:

X_{d_k} = ha of land for crop k with planting date d_k;

$l_{i,k}$ = selected insurance policy for crop k (binary), where $l_{i,k} = 1$ if farmer selects policy i for crop k, otherwise $l_{i,k} = 0$.

The following equalities are valid, $\sum_i l_{i,k} = 1$, $k = 1, 2, \ldots, K$ because the farmer buys only one insurance policy for each crop $k = 1, 2, \ldots, K$.

Table 1. Model parameters.

Variable	Unit	Description
C_k	\$ ha^{-1}	Production cost of crop k .
$R_{i,k}$	\$ ha^{-1}	Premium of the insurance policy i for crop k .
P_k^s	\$ kg^{-1}	Market price of crop k in scenario s .
P_k^*	\$ kg^{-1}	Price election of crop k , i.e., the expected market price. This price is set by FCIC (Federal Crop Insurance Corporation) before the sales closing date for the crop.
$y_{d_k}^s$	kg ha^{-1}	Yield of crop k per ha for planting date d_k in scenario s .
$y_{i,k}^*$	kg ha^{-1}	Insured yield of crop k per ha by policy i .

2.2 Objective

The objective is to minimize a farmer's expected losses, which is equivalent to maximizing the expected revenue. Variable cost per crop is composed of the production cost and the insurance premium cost. Total revenue includes the revenue from selling of actual yield and that from the insurance indemnity, if received.

Y_k^s is the total yield of crop k in scenario s , i.e., $Y_k^s = \sum_{d_k} X_{d_k} y_{d_k}^s$. Let $Z_{i,k}^s$ be the difference between the insured yield and the true yield, $Z_{i.k}^s = \sum_{d_k} X_{d_k} (y_{i,k}^* - y_{d_k}^s)$, thus the indemnity yield is $(Z_{i,k}^s)^+$. The loss function equals $f(\bar{x}, \vec{x}) = \sum_{k=1}^{K} \{C_k q_k - Y_k^s P_k^s + \sum_{i=1}^{I} l_{i,k} [R_{i,k} q_k - (Z_{i,k}^s)^+ P_k^*]\}$.

Substituting Y_k^s and $Z_{i,k}^s$ to the loss function gives:

$$f(\bar{x}, \vec{x}) = \sum_{k=1}^{K} \{C_k q_k - (\sum_{d_k} X_{d_k} y_{d_k}^s) P_k^s + \sum_{i=1}^{I} l_{i,k} [R_{i,k} q_k - (\sum_{d_k} X_{d_k} (y_{i,k}^* - y_{d_k}^s))^+ P_k^*]\} ,$$

where: $\bar{x} = \{X_{d_k}, l_{i,k}\}$ is the decision vector, $\vec{x} = \{Y_k^s, P_k^s\}$ is the random vector.

We minimize the expected cost: $\min E(f(\bar{x}, \vec{x}))$.

2.3 Constraints

The most significant constraint is the CVaR constraint. Figure 1 shows a simple illustration of CVaR, By definition, CVaR is the average of values exceeding a selected percentile, a, of a random variable. The a-percentile is called VaR in finance applications.

A farmer can control the expected loss exceeding VaR and assure that it is less than a certain threshold value v. This is modeled by CVaR constraint:

$$CVaR_a[f(\bar{x},\vec{x})] \leq v \quad ,$$

where: $f(\bar{x},\overline{x})$ is a loss function, and $a = \Pr[f(\bar{x},\vec{x}(w)) \leq VaR]$ is the confidence level. If a farm grows q_k ha of crop k and every crop has d_k planting dates; then we must have:

$$\sum_{d_k} X_{d_k} = q_k \text{ and } X_{d_k} \geq 0, \text{ for } k = 1,2\ldots K.$$

A farmer can buy either no insurance or one type of insurance policy for every crop, consequently:

$$\sum_i l_{i,k} = 1, \text{ for } k = 1,2\ldots K$$

where $?_{i,k}$’s are binary numbers.

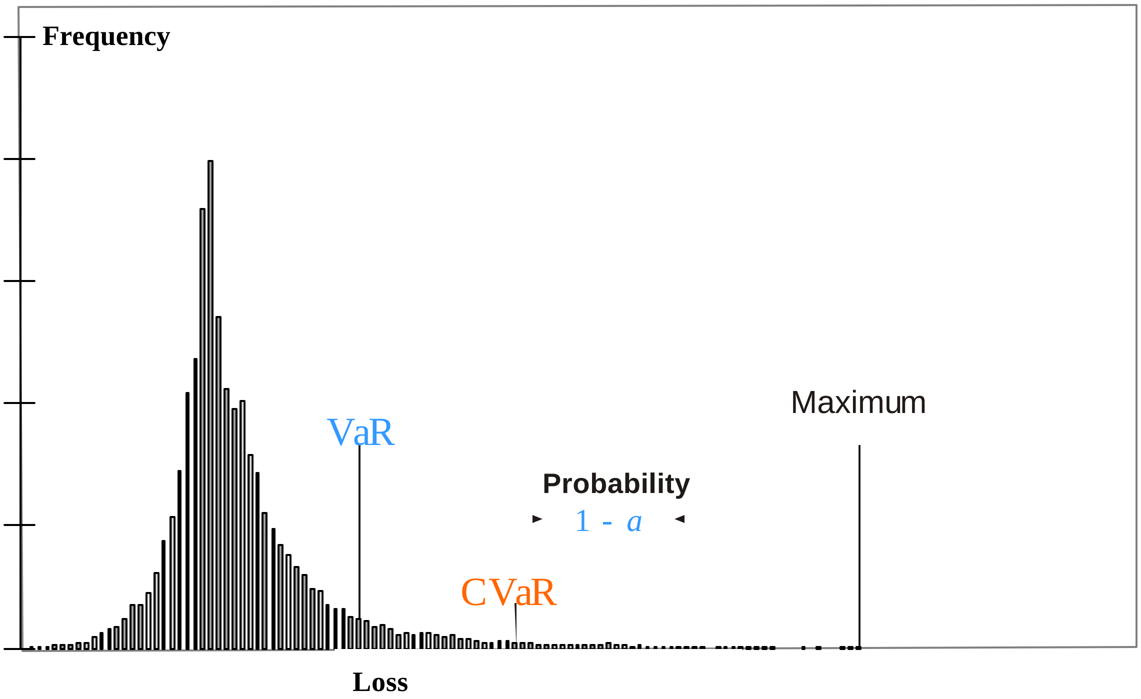

Figure 1. Loss distribution, a-VaR, and a-CVaR.

2.4 Complete model formulation

The optimization problem can be expressed as:

$$\mathrm{Min}\, E(f(\vec{x},\overrightarrow{x}))$$

$$\text{s.t.} \quad f(\vec{x},\overrightarrow{x}) = \sum_{k=1}^{K} \{C_k q_k - Y_k^s P_k^s + \sum_{i=1}^{I} l_{i,k}[R_{i,k} q_k - (Z_{i,k}^s)^+ P_k^*]\}$$

$$Y_k^s = \sum_{d_k} X_{d_k} y_{d_k}^s$$

$$Z_{i.k}^s = \sum_{d_k} X_{d_k} (y_{i,k}^* - y_{d_k}^s)$$

$$\sum_{d_k} X_{d_k} = q_k \text{ and } X_{d_k} \geq 0, \text{ for } k = 1,2 \ldots K$$

$$\sum_{i} l_{i,k} = 1, \text{ for } k = 1,2 \ldots K\text{, where } l_{i,k} \text{ are binary numbers}$$

$$CVaR_a[f(\vec{x},\overrightarrow{x})] \leq v$$

3. Case Study

We used the same dataset as in the case study of Cabrera *et al.* (2005b). We optimized a 40-ha non-irrigated farm in Jackson County, FL that allocates half of its land to cotton and the other half to peanut. For cotton, there were four planting dates: 16 and 23 April and 1 and 8 May. For peanut, there were nine planting dates, two dates in April, five in May, and two in June.

The three main types of crop insurances are: 1) Actual Production History (APH) or Multi-Peril Crop Insurance (MPCI); 2) Crop Revenue Coverage (CRC); and 3) Catastrophic Coverage (CAT). APH assures a percentage of the farmers' historic yield. If the crop yields less than the insured percentage, the insurance pays an indemnity covering the difference between the insured percentage and the yield loss. CRC assures income by indemnifying farmers based on historical yield and a pre-fixed market price. If actual yield multiplied by actual market price is lower than an indemnified income level, the farmer is entitled to an indemnity payment. CAT can be defined as an APH policy at 50% yield coverage with 55% price base election. Crop insurance products included the most popular contracts are listed in Table 2.

A farmer can choose either no insurance or one level of three types of insurance products for each crop. Including the "no insurance" option, there were five options for peanut and 10 for cotton. There were in total fifty possible selections of crop insurance combinations for cotton and peanut. The price of the insurance premium depends on the type of the policy, coverage level, location and historical yield, which were estimated using the premium calculator from the Risk

Management Agency (http://www3.rma.usda.gov/apps/premcalc/). We used 100% of the price election for APH and CRC crop insurance products as they are the most common choices.

Table 2. Crop insurance products, coverage levels, premium prices, and average yields used in the farm model analysis. Source: Cabrera et al. (2005b).

	Peanut	Cotton
Actual Production History in 5% increments	65 to 75%	65 to 75%
Crop Revenue Coverage in 5% increments	N/A	65 to 85%
Price Base 2004, \$ kg $^{-1}$	0.3935	1.4991
APH Premium Range in 5% increments	9.64-41.27	21.50-93.90
CRC Premium Range in 5% increments	N/A	27.18-288.87
Average yield, Mg ha $^{-1}$	3.362	0.729

Crops yields were simulated using the models available in the Decision Support System for Agrotechnology Transfer (DSSAT) v4.0 (Jones et al., 2003), namely CROPGRO-Peanut (Boote et al., 1998) and CROPGRO-Cotton (Messina et al., 2005). The models were calibrated and tested for the management practices and environmental conditions in the southeastern U.S. (Mavromatis et al., 2002; Messina et al., 2005). Crop model simulations used current management practices in the region for varieties, fertilization, and planting dates, and the representative soil type *Dothan Loamy Sand* (Cabrera et al., 2005). In the case of peanut, the most widely planted variety in the region, Georgia Green, was used for the simulations. It is a runner type variety with medium maturity and moderate resistance to tomato spotted wilt virus and to *Cylindrocladium* black rot. For cotton, a popular medium to full season variety from Delta & Pine Land ® was used, DP 555.

We simulated yields of cotton and peanut using historic climate data between 1939 and 2004 (65 years) categorized by ENSO phase. We also used simulated market prices of the two crops for the same set of years based on 10 years (1996-2005) of historical records from the National Agricultural Statistical Service from the US Department of Agriculture (http://www.nass.usda.gov) and ENSO phases (JMA, 1991). We used Matlab 7.01 to perform optimizations.

3.1 Model results without CVaR constraint

3.1.1 Optimal insurance choices

The model results without CVaR constraint are shown in Figure 2. For Neutral and El Niño years, buying no insurance for cotton and 75%APH for peanut is the optimal solution. With these insurance products, revenue is $16,250 for Neutral years and $17,657 for El Niño years. For La Niña years, buying no insurance for cotton and 65%APH for peanut is the optimal solution with revenue of $16,315. The curve of all years in Figure 2 shows the result of optimizing without distinguishing among ENSO phases. Revenues for all years are less than those from using ENSO-based information, which demonstrates the value of climate information. The optimal solution forall years is buying no insurance for cotton and 75%APH for peanut, which is the same as for Neutral and El Niño years. With about 50% of years being Neutral and 25% being El Niño, it is not surprising that the optimal insurance solution for all years would be the same as that for the majority of years by ENSO phase.

Figure 3 shows the distribution of revenues based on the best crop insurance selection for three ENSO phases. For example, the figure shows that the probability of getting $20,000 revenue is approximately 0.17 for Neutral, 0.06 for El Niño, and 0.14 for La Niña years.

3.1.2 Optimal planting dates

Only one optimal planting date is selected for each crop insurance contract and ENSO phase. For peanut, the best planting dates are May 22 in El Niño years and May 29 in La Niña and Neutral years; for cotton, the best planting dates are April 16 in Neutral years, May 1 in La Niña years, and May 8 in El Niño years.

3.1.4 Results without a no insurance option

Lenders and policy makers usually urge farmers to buy at least some type of crop insurance. If a farmer must purchase at least some insurance product, the optimal insurance contract for cotton would change to 75% APH in all ENSO phases. The optimal planting dates remain the same.

3.2 Results with CVaR Constraint

The complete 95% CVaR model results for all ENSO phases are shown in Table 3. Taking La Niña years as an example, if a farmer accepts that 95% CVaR of the loss is less than $11,000, he should purchase 65%APH for peanut and no insurance for cotton, but if a farmer wants to limit 95% CVaR by $6,000, with the best choice is 75%APH for peanut and no insurance for cotton. A farmer can reduce risk by reducing the expected revenue.

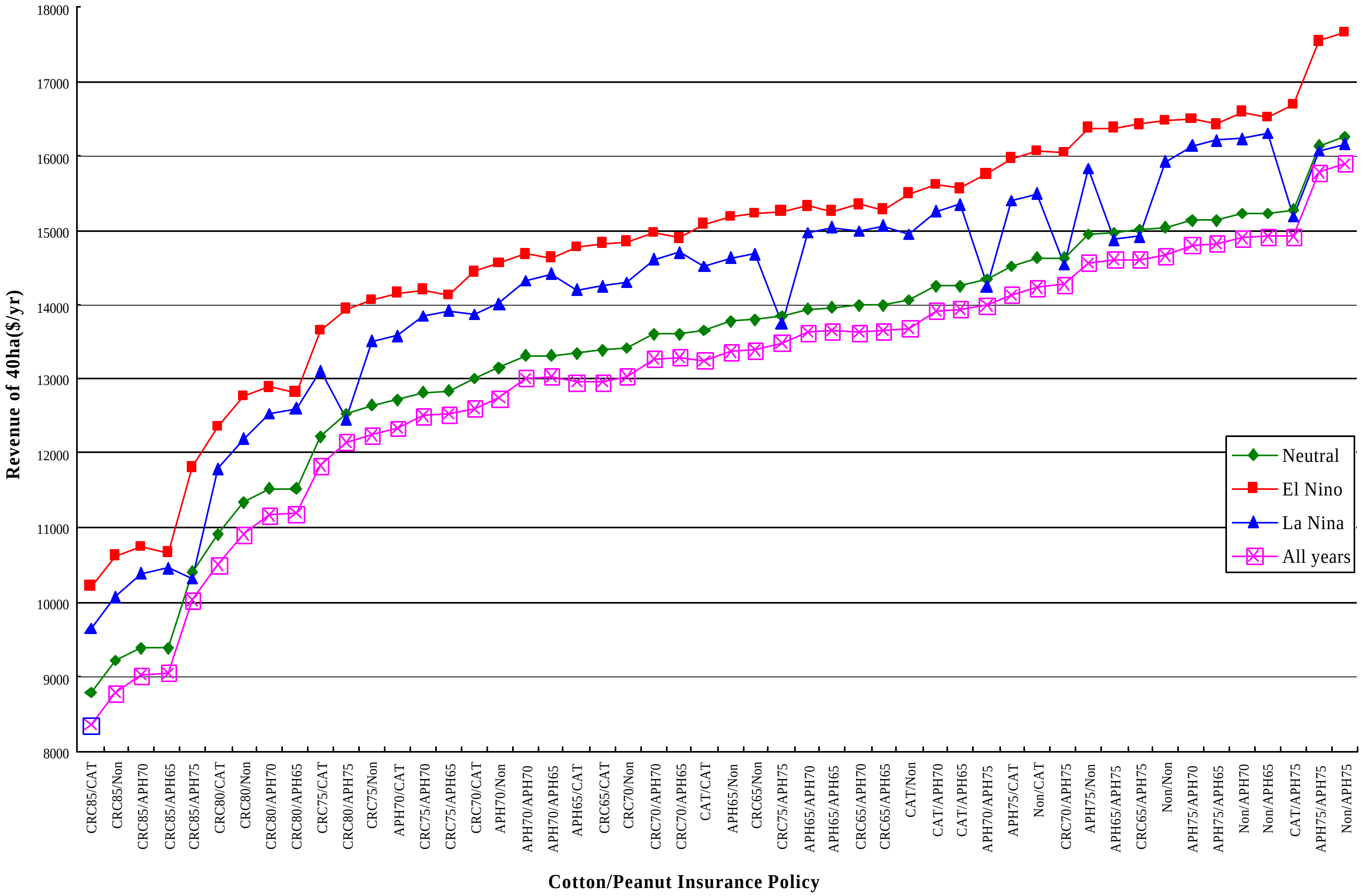

Figure 2. Optimal revenue by crop insurance product and ENSO phase without CVaR constraints. APH65/CRC80 means APH 65% for cotton and CRC 80% for peanut.

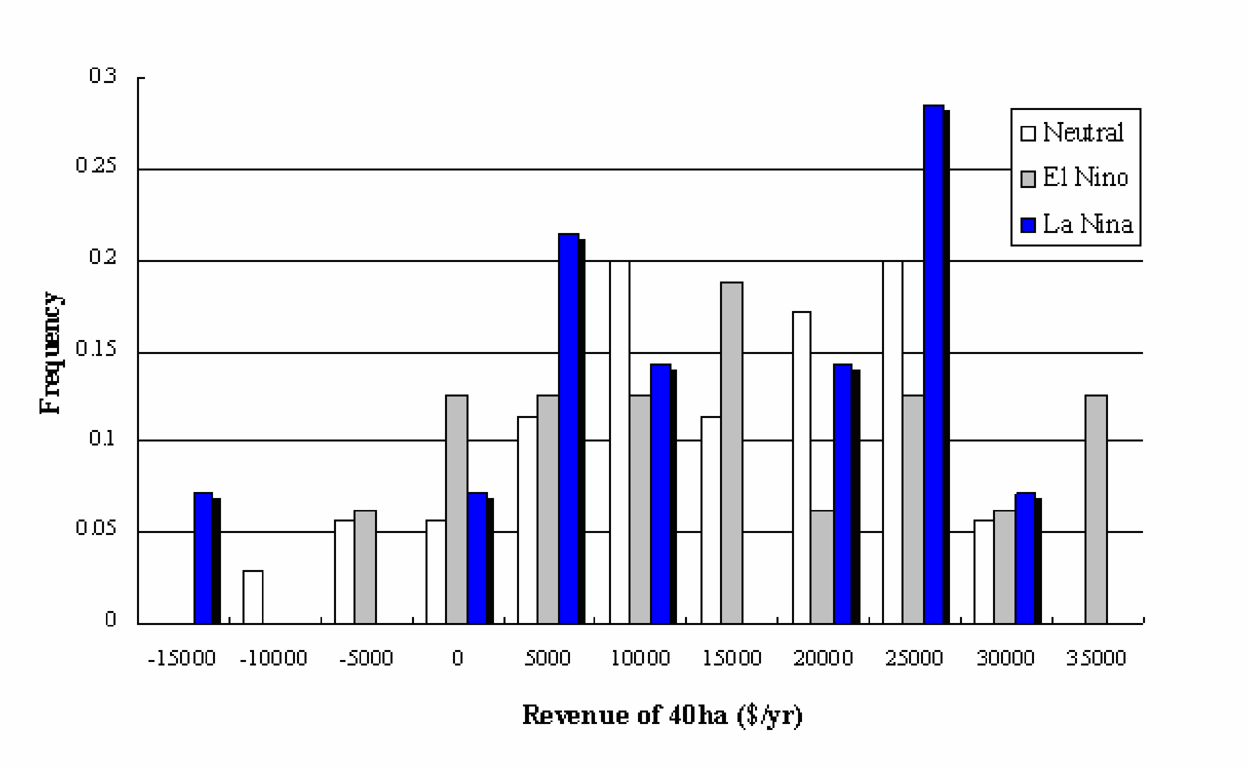

Figure 3. Distribution of optimal income for all ENSO phases without CVaR constraint. APH65/CRC80 means APH 65% for cotton and CRC 80% for peanut.

Table 3. CVaR model at 95% for all ENSO phases.

ENSO Phase	95%CVaR limit *v*	Expected revenue	Optimal Insurance	Optimal Planting Date
Neutral	$6,827 and above	$16,250	Cotton: No insurance Peanut: 75%APH	Cotton: April 16 Peanut: May 29
	Below $6,827	infeasible	infeasible	infeasible
El Niño	$3,717 and above	$17,657	Cotton: No insurance Peanut: 75%APH	Cotton: May 8 Peanut: May 22
	Below $3,717	infeasible	infeasible	infeasible
La Niña	$10,624 and above	$16,315	Cotton: No insurance Peanut: 65%APH	Cotton: May 1 Peanut: May 29
	Between $9,559 and $10,624	$16,235	Cotton: No insurance Peanut: 70%APH	
	Between $5,814 And $9,559	$16,158	Cotton: No insurance Peanut: 75%APH	
	Below $5,814	infeasible	infeasible	infeasible

If a farmer is required to buy at least some insurance contract for all crops the no insurance option for cotton in Table 3 would be replaced by 75% APH.

A farmer will have three possible combinations of insurance choices during La Niña years depending on the risk aversion level. Figure 4 compares the distribution of the revenues by those three insurance combinations.

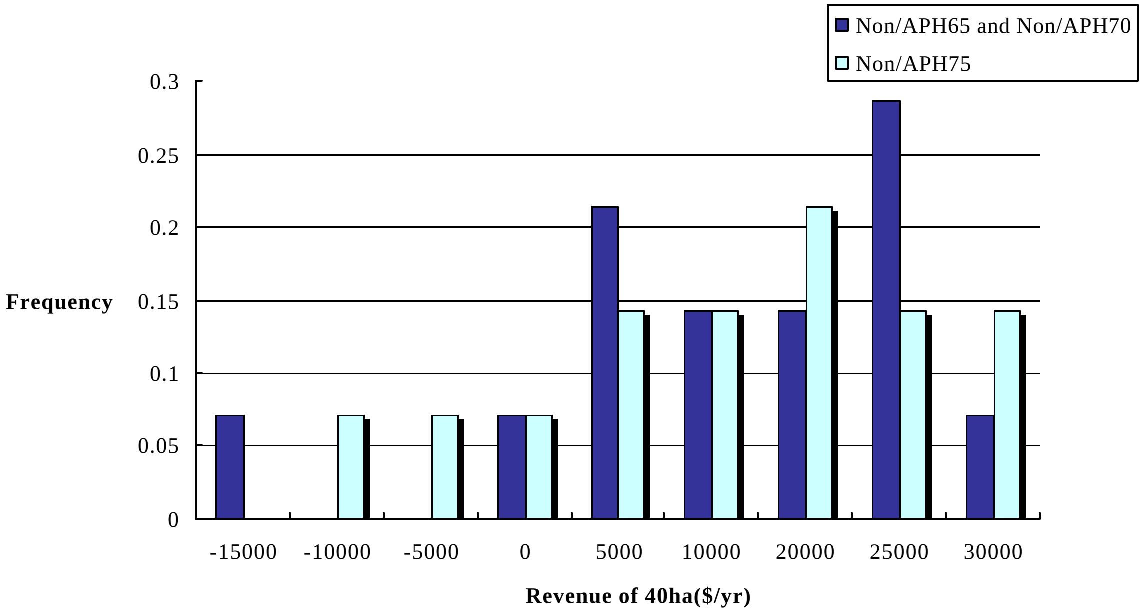

Figure 4: Distribution of optimal revenue for La Niña years under different 95% CVaR limit values.

4. Conclusion

This research studied the impact of climate variability and uncertain prices on crop insurance decisions. We created a stochastic model to select optimal crop insurance products according to forecasts of ENSO phase. The model solves a stochastic optimization problem with CVaR constraints. Taking advantage of the ENSO-based climate forecasts, the model can select optimal crop insurance products.

We illustrated the model with a case study in north Florida farm growing cotton and peanut. Results showed that optimum insurance vared with ENSO-based climates forecasts and risk levels specified by CVaR. For a risk neutral farmer, buying no insurance for cotton and 75% APH for peanut was the optimal solution for Neutral and El Niño years, and buying no insurance for cotton and 65%APH for peanut was optimal for La Niña years. The insurance strategy for cotton in La Niña years changed to 70% APH for a risk averse farmer and to 75% APH for a highly risk averse farmer. If farmer is required to have at least some type of crop insurance for both crops, the best selection for cotton would be 75% APH.

Results of this study are consistent with findings of Cabrera et al. (2005b). They found that optimal policy is no insurance for cotton and 75% APH for peanut during all ENSO phases in a risk neutral case. Also, they found that it was optimal for peanut farmers to purchase 70% APH for El Niño and Neutral years, and 65% APH for La Niña years. However, they found CAT to be the next best option for cotton if farmers are required to have at least some insurance contract.

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